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$9^n + 9^{n-1} + 9^{n-2} + \cdots + 9^2 + 9$. Altogether, therefore, a_{n+1} contains less than $8(9^n + 9^{n-1} + 9^{n-2} + \cdots + 9^2 + 9) < 9^{n+1}$ terms of I, and, as each of these terms is not greater than $1/10^n$, we have $a_{n+1} < (9^{n+1}/10^n)$.

Our series is therefore

$$s = a_1 + a_2 + a_3 + \cdots < 9 + \frac{9^2}{10} + \frac{9^3}{10^2} + \cdots + \frac{9^{n+1}}{10^n} + \cdots = 90,$$

and I is certainly convergent.

The preceding proof also contains all material necessary to show the following:

Let M be any positive integer, and N_1 the number of positive integers $< M$ containing no figure 9, N_2 the number of positive integers $< M$ containing at least one figure 9, then

$$\lim_{M \doteq \infty} N_1/N_2 = 0.$$

See problem 207 proposed on page 55 of this issue.

THE PERFECT MAGIC SQUARES FOR 1914.

By V. M. SPUNAR, Chicago, Ill.

Using Zerr's formula*

$$\frac{1914 - (n^3 + n)/2}{n} = \text{Integer},$$

the only values of n for 1914 are 3, 11, 29. (For 1913 there are no values for n whatever.) The least and greatest integers permissible for forming the squares are represented by another formula of Zerr's, namely,

$$\frac{3828 \pm (n^3 - n)}{2n}.$$

For $n = 3$, the least integer = 634, the greatest integer = 642. For $n = 11$, the least integer = 114, the greatest integer = 234. For $n = 29$, the least integer = − 354, the greatest integer = 486.

Hence the magic squares for 1914 with only positive numbers are as follows:

$n = 3$

641	634	639
636	638	640
637	642	635

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* American Mathematical Monthly, Vol. XVI, no. 1, page 2.

$n = 11$

181	194	207	220	233	114	127	140	153	166	179
193	206	219	232	124	126	139	152	165	178	180
205	218	231	123	125	138	151	164	177	190	192
217	230	122	135	137	150	163	176	189	191	204
229	121	134	136	149	162	175	188	201	203	216
120	133	146	148	161	174	187	200	202	215	228
132	145	147	160	173	186	199	212	214	227	119
144	157	159	172	185	198	211	213	226	118	131
156	158	171	184	197	210	223	225	117	130	143
168	170	183	196	209	222	224	116	129	142	155
169	182	195	208	221	234	115	128	141	154	167

NOTE ON A MEMORY DEVICE FOR HYPERBOLIC FUNCTIONS.

By F. S. ELDER, Central High School, Kansas City, Mo.

Purely as a memory aid for expressing any hyperbolic function $\phi h(u)$ in terms of any other hyperbolic function $\psi h(u)$, I offer the following method which I never have seen in print.

Draw the customary right triangle ABC right angled at C, as for the circular functions. Write $\psi h(u)$ and unity upon the same sides of the triangle as you would to show the corresponding circular function. Then take the triangle by the point C and deform it until B becomes the right angle. In this deformation, the two sides on which the numbers are written are supposed to retain their original length, while the third side is deformed (lengthened or shortened) and likewise in returning to the original form. Now compute the third side in terms of the other two and let the triangle recover its original shape. All the other hyperbolic functions, $\phi h(u)$, can now be written down directly in terms of $\psi h(u)$ by means of the same ratios of the sides of the original triangle that give the corresponding circular functions.

Examples: (1) If $\psi h(u)$ be $\tan h(u)$, write $AC = 1$, $BC = \tan h(u)$. Deform. Then $AB = \sqrt{1 - \tan^2 h(u)}$; now restore the triangle.

Then

$$\sin h(u) = BC/AB = \frac{\tan h(u)}{\sqrt{1 - \tan^2 h(u)}}, \quad \sec h(u) = AB/AC = \sqrt{1 - \tan^2 h(u)}, \text{ etc.}$$

(2) If $\psi h(u)$ be $\csc h(u)$, write $AB = \csc h(u)$; $BC = 1$. Deform. Then $AC = \sqrt{1 + \csc^2 h(u)}$. Restore.

Then

$$\cos h(u) = AC/AB = \frac{\sqrt{1 + \csc^2 h(u)}}{\csc h(u)}, \quad \tan h(u) = BC/AC = \frac{1}{\sqrt{1 + \csc^2 h(u)}}, \text{ etc.}$$